

Review 21 properties of definite integrals
 average value of a function
 MVT for definite integrals

Properties

$$\int_a^b f(x) dx = - \int_b^a f(x) dx$$

$$\int_a^b k \cdot f(x) dx = k \int_a^b f(x) dx$$

$$\int_a^a f(x) dx = 0$$

$$\int_a^b f(x) dx + \int_b^c f(x) dx = \int_a^c f(x) dx$$

Apr 5-8:57 AM

Ex 1. $\int_2^5 f(x) dx = 12$ $\int_5^8 f(x) dx = 4$

a) $\int_2^8 f(x) dx = 16$

b) $\int_5^2 f(x) dx = -12$

c) $\int_2^6 f(x) dx + \int_6^8 f(x) dx = 16$

d) $\int_2^5 f(x) dx - \int_5^8 f(x) dx = 8$

Apr 5-10:03 AM

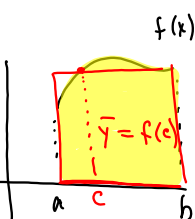
average value of a function

$$\bar{y} = \frac{1}{b-a} \int_a^b f(x) dx$$

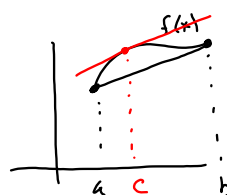
$$(b-a) \bar{y} = \int_a^b f(x) dx$$

area of rectangle = area under curve

MVT says c exists



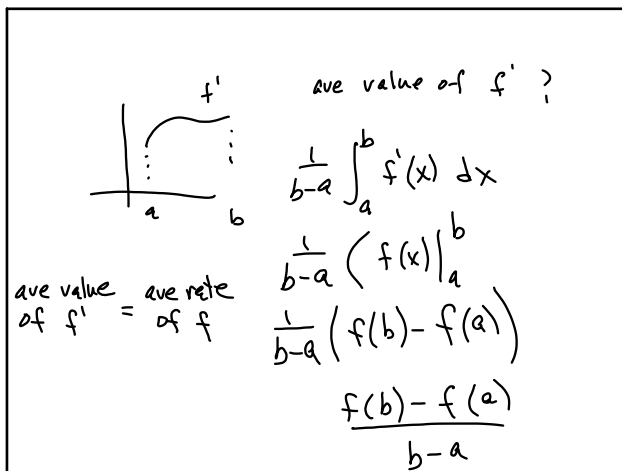
MVT for derivatives says c exists



$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Apr 5-10:07 AM

Apr 5-10:14 AM



Apr 5-10:17 AM

Ex 2. Water pours into a tank
at a rate of $f(t) = t^2 \frac{\text{gal}}{\text{min}}$.

What is the average rate of water flow
on $[0, 4]$

$$\frac{1}{4-0} \int_0^4 t^2 dt = \frac{1}{4} \left. \frac{t^3}{3} \right|_0^4 = \frac{1}{4} \left(\frac{4^3}{3} - 0 \right) = \frac{16}{3}$$

$$\text{Water} = \frac{t^3}{3} + C$$

$$\text{ave rate of water on } [0, 4] = \frac{\left(\frac{4^3}{3} + C \right) - \left(\frac{0^3}{3} + C \right)}{4-0} = \frac{16}{3}$$

Apr 5-10:21 AM